

CBCS SCHEME

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18MAT11

First Semester B.E. Degree Examination, Aug./Sept.2020 Calculus and Linear Algebra

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. With usual notation, prove that for the curve $r = f(\theta)$, $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4} \left(\frac{dr}{d\theta} \right)^2$. (06 Marks)
- b. Find the radius of curvature at any point P(x, y) on the parabola $y^2 = 4ax$. (06 Marks)
- c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)^3$. (08 Marks)

OR

- 2 a. Find the pedal equation of the curve $\frac{2a}{r} = 1 - \cos\theta$. (06 Marks)
- b. Find the radius of curvature of the tractrix $x = a[\cos t + \log \tan(t/2)]$, $y = a \sin t$. (06 Marks)
- c. Show that the angle between the pair of curves: $r = 6\cos\theta$ and $r = 2(1 + \cos\theta)$ is $\pi/6$. (08 Marks)

Module-2

- 3 a. Using Maclaurin's series, prove that $\sqrt{1 + \sin 2x} = 1 + x - x^2/2! - x^3/3! + x^4/4!$ (06 Marks)
- b. Evaluate: i) $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{1/x}$ ii) $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x}{2} \right)^{1/x}$ (07 Marks)
- c. Examine the function $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ for its extreme values. (07 Marks)

OR

- 4 a. If $U = f(x - y, y - z, z - x)$ show that $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z} = 0$. (06 Marks)
- b. If $u = x \cos y \cos z$, $v = x \cos y \sin z$, $w = x \sin y$, then show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = -x^2 \cos y$. (07 Marks)
- c. Find the volume of the largest rectangular parallelepiped that can be inscribed in the Ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. (07 Marks)

Module-3

- 5 a. Evaluate $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dz dy dx$ (06 Marks)
- b. Find the volume of the solid bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + y + z = 1$ (07 Marks)
- c. Show that $\beta(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}$ (07 Marks)

Important Note: 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and/or equations written eg, 42-8=50, will be treated as malpractice.

OR

- 6 a. Change the order of Integration and hence evaluate

$$\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} xy \, dy \, dx$$

(06 Marks)

- b. Find the centre of gravity of the curve
- $r = a(1 + \cos\theta)$
- .

(07 Marks)

- c. Prove that
- $\int_0^{\pi/2} \sqrt{\sin\theta} \, d\theta \cdot \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$

(07 Marks)

Module-4

- 7 a. A body in air at 25°C cools from 100°C to 75°C in 1 minute. Find the temperature of the body at the end of 3 minutes. (06 Marks)
- b. Find the orthogonal trajectories of the family of cardioids $r = a(1 + \cos\theta)$ (07 Marks)
- c. Solve: $[4x^3y^2 + y\cos(xy)]dx + [2x^4y + x\cos(xy)]dy = 0$ (07 Marks)

OR

- 8 a. A series circuit with resistance R , inductance L and electromotive force E is governed by the differential equation $L \frac{di}{dt} + Ri = E$, where L and R are constants and initially the current i is zero. Find the current at any time t . (06 Marks)
- b. Solve: $x^3 \frac{dy}{dx} - x^2y = -y^4 \cos x$ (07 Marks)
- c. Solve: $x^2p^2 + xp - (y^2 + y) = 0$, where $p = \frac{dy}{dx}$. (07 Marks)

Module-5

- 9 a. Find the rank of the matrix $\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by applying elementary row operations. (06 Marks)
- b. Find the dominant Eigen value and the corresponding Eigen vector of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by Rayleigh's power method taking the initial Eigen vector as $[1, 1, 1]^T$. (07 Marks)
- c. Apply Gauss-Jordan method to solve the system of equations: $2x + 5y + 7z = 52$, $2x + y - z = 0$, $x + y + z = 9$. (07 Marks)

OR

- 10 a. Test for consistency and solve: $5x_1 + x_2 + 3x_3 = 20$, $2x_1 + 5x_2 + 2x_3 = 18$, $3x_1 + 2x_2 + x_3 = 14$ (06 Marks)
- b. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form. (07 Marks)
- c. Solve the system of equations $5x + 2y + z = 12$
 $x + 4y + 2z = 15$
 $x + 2y + 5z = 20$
 Using Gauss-Siedel method [carry out 4 iterations]. (07 Marks)
